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PALM DISTRIBUTION AND LIMIT THEOREMS FOR RANDOM POINT PROCESSES IN R^n

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1. Introduction. The paper deals with homogeneous random point processes in R^n. Our aim is to obtain sufficient conditions for asymptotic normality of the number of points of a point process P in a ball when the radius tends to infinity. These conditions are formulated in terms of Palm distributions of the point process P. The proofs of the theorems stated below are based on the relation between the distribution P of a point process in R^n and its Palm distribution due to Ambartzumian [1]. In the one-dimensional case these relations reduce to the so-called Palm-Khinchin formulae [2].

2. Notation. Let M be the class of all countable subsets of R^n such that any $m \in M$ has no cluster point in a bounded subset of R^n.

Define $N(B, m)$ to be the number of points in $B \cap m$, where B is a bounded Borel set in R^n and $m \in M$.

Denote by C the minimal σ-algebra of subsets of M containing all subsets of the form $\{m\colon N(B, m) = k\}$, $k = 0, 1, 2, \ldots$ Any probability measure P on C describes a random point process.

A random point process P is said to be *homogeneous* if, for any $c \in C$, $P(tc)$ does not depend on $t \in T$, where T denotes the group of all translations of R^n and

$$tc = \{m\colon t^{-1} m \in c\}.$$

Further, we assume that for every bounded Borel set B in R^n

$$\mathrm{E}_P(N(B)) = \lambda |B|, \quad \lambda < \infty, \tag{1}$$

where $|B|$ is the volume of B. In other words, we consider the finite intensity case.

3. Main results. Let $S(v)$ be the sphere of volume v centred at the origin in R^n. We consider the random number $N(S(v))$ of points of the process in $S(v)$ for large values of v.

$N(S(v))$ is called *asymptotically normal* if

$$\sum_{k:(k-\lambda v)/\sqrt{\lambda v}<\alpha} P(N(S(v))=k) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} \exp\left\{-\frac{s^2}{2}\right\} ds \quad \text{as } v\to\infty,\ \alpha\in R^1. \tag{2}$$

$N(S(v))$ is called *locally asymptotically normal in variation* if

$$\sum_{k=0}^{\infty} \left| P(N(S(v))=k) - \frac{1}{\sqrt{2\pi\lambda v}} \exp\left\{-\frac{(k-\lambda v)^2}{2\lambda v}\right\}\right| \to 0 \quad \text{as } v\to\infty. \tag{3}$$

$N(S(v))$ is called *locally asymptotically normal* if

$$\sup_k \left| \sqrt{\lambda v}\, P(N(S(v))=k) - \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{(k-\lambda v)^2}{2\lambda v}\right\}\right| \to 0 \quad \text{as } v\to\infty. \tag{4}$$

Note that each of the relations (3) and (4) implies (2). It is known that under the assumption (1) the limit

$$\pi_k(v) = \lim_{h\to 0} \frac{P(\{N(S(v))=k\} \cap \{N(S(v+h)-S(v))=1\})}{P(N(S(v+h)-S(v))=1)} \tag{5}$$

exists [1].

We call this limit the *spherical Palm distribution.* This limit can be interpreted as the conditional probability of $\{N(S(v))=k\}$ under the condition that a point of the random process P lies on the boundary of the sphere $S(v)$. By the *Palm distribution* we usually mean the conditional probability of $\{N(S(v))=k\}$ under the condition that there is a point of the process P at the origin. The spherical Palm distribution can be found by integration of the usual Palm distribution over the boundary of the sphere $S(v)$ (see [1]).

The conditions for asymptotic normality will be given in terms of the variational distance

$$\varrho(v) = \sum_{k=0}^{\infty} |P_k(v) - \pi_k(v)|, \qquad P_k(v) = P(N(S(v))=k). \tag{6}$$

We show that the above-mentioned types of asymptotic normality of $N(S(v))$ are implied by various assumptions concerning the rate of convergence of $\varrho(v)$ to zero as v tends to infinity. In this sense the condition on the rate of convergence of $\varrho(v)$ to zero can replace the usual mixing condition [3].

In the sequel we prove the following theorems:

THEOREM 1. *If*

$$\lim_{v\to\infty}\frac{1}{\sqrt{v}}\int_0^v \varrho(u)\,du = 0,$$

then $N(S(v))$ is asymptotically normal.

THEOREM 2. *If*

$$\int_0^\infty \varrho(u)\,du < \infty,$$

then $N(S(v))$ is locally asymptotically normal in variation.

THEOREM 3. *If*

$$\lim_{v\to\infty}\frac{1}{\sqrt{v}}\int_0^v u\varrho(u)\,du = 0,$$

then $N(S(v))$ is locally asymptotically normal.

The proofs of these theorems are based on the following Ambartzumian relations (see [1]):

$$\frac{dP_0(v)}{dv} = -\lambda\pi_0(v),$$

(7)

$$\frac{dP_k(v)}{dv} = -\lambda\big(\pi_k(v)-\pi_{k-1}(v)\big), \quad k = 1, 2, \ldots,$$

$$P_k(0) = \begin{cases} 1 & \text{for } k = 0, \\ 0 & \text{for } k \neq 0. \end{cases}$$

Further, without loss of generality we can assume that $\lambda = 1$.

In the one-dimensional case, Theorems 1 and 2 were announced in [4].

The role of the Palm distribution in the problems related to asymptotic normality of $N(S(v))$ was first noticed by R. V. Ambartzumian to whom Theorem 1 should be attributed.

The author expresses his gratitude to Professor R. V. Ambartzumian for suggesting the present topic.

4. Proof of Theorem 1. Rewrite (7) in the form

$$\frac{dP_0(v)}{dv} = -P_0(v)+\big(P_0(v)-\pi_0(v)\big),$$

(8)

$$\frac{dP_k(v)}{dv} = -\big(P_k(v)-P_{k-1}(v)\big)+\big(P_k(v)-\pi_k(v)\big)+\big(\pi_{k-1}(v)-P_{k-1}(v)\big),$$

$$k = 1, 2, \ldots$$

We introduce the generating functions

$$\Pi_z(v) = \sum_{k=0}^{\infty} P_k(v) z^k, \qquad A_z(v) = \sum_{k=0}^{\infty} (\pi_k(v) - P_k(v)) z^k$$

for which from (8) we derive

$$\frac{d\Pi_z(v)}{dv} = (z-1)\Pi_z(v) + (z-1)A_z(v), \qquad \Pi_z(0) = 1. \tag{9}$$

Hence we get

$$\Pi_z(v) = e^{(z-1)v} + (z-1)e^{(z-1)v} \int_0^v e^{(1-z)u} A_z(u)\,du. \tag{10}$$

The characteristic function of the distribution $\{P_k(v)\}$ can be obtained by substituting z by e^{it}. Hence it is enough to show that for fixed t

$$\exp\{-it\sqrt{v}\}\,\Pi_{\exp\{it/\sqrt{v}\}}(v) \to \exp\{-t^2/2\} \quad \text{as } v \to \infty.$$

Since the Poisson distribution is asymptotically normal, it remains to show that the contribution of the second summand in (10) vanishes as v tends to infinity.

We have

$$\left|\exp\{-it\sqrt{v}\}\left(\exp\left\{\frac{it}{\sqrt{v}}\right\}-1\right)\times\right.$$

$$\left.\times\int_0^v \exp\left\{\left[\exp\left(\frac{it}{\sqrt{v}}\right)-1\right](v-u)\right\} A_{\exp\{it/\sqrt{v}\}}(u)\,du\right|$$

$$\leqslant \left|\exp\left\{\frac{it}{\sqrt{v}}\right\}-1\right| \int_0^v \left|\exp\left\{\left[\exp\left(\frac{it}{\sqrt{v}}\right)-1\right](v-u)\right\}\right| |A_{\exp\{it/\sqrt{v}\}}(u)|\,du$$

$$\leqslant \frac{t}{\sqrt{v}} \int_0^v \exp\left\{\left(\cos\frac{t}{\sqrt{v}}-1\right)(v-u)\right\} |A_{\exp\{it/\sqrt{v}\}}(u)|\,du.$$

Since

$$[\cos(t/\sqrt{v})-1](v-u) \leqslant 0 \text{ for } u \in (0, v) \quad \text{and} \quad |A_{\exp\{it/\sqrt{v}\}}(u)| \leqslant \varrho(u),$$

the last expression does not exceed

$$\frac{t}{\sqrt{v}} \int_0^v \varrho(u)\,du.$$

Hence Theorem 1 holds.

5. Proof of Theorem 2. We first show that under the assumptions of Theorem 2 we have

$$\sum_{k=0}^{\infty} |P_k(v) - A_k(v)| \to 0 \quad \text{as } v \to \infty,$$

where

$$\Lambda_k(v) = e^{-v}\frac{v^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

Since the probabilities $\Lambda_k(v)$ satisfy the equations

$$(11) \qquad \begin{aligned} \frac{d\Lambda_0(v)}{dv} &= -\Lambda_0(v), \\ \frac{d\Lambda_k(v)}{dv} &= -\big(\Lambda_k(v) - \Lambda_{k-1}(v)\big), \qquad k = 1, 2, \ldots, \end{aligned}$$

$$\Lambda_k(0) = \begin{cases} 1 & \text{for } k = 0, \\ 0 & \text{for } k \neq 0, \end{cases}$$

using (7) we can write

$$(12) \qquad \begin{aligned} \frac{d}{dv}\big(P_0(v) - \Lambda_0(v)\big) &= -\big(P_0(v) - \Lambda_0(v)\big) - \big(\pi_0(v) - P_0(v)\big), \\ \frac{d}{dv}\big(P_k(v) - \Lambda_k(v)\big) &= \big(P_{k-1}(v) - \Lambda_{k-1}(v)\big) - \big(P_k(v) - \Lambda_k(v)\big) + \\ &\quad + \big(\pi_{k-1}(v) - P_{k-1}(v)\big) - \big(\pi_k(v) - P_k(v)\big), \qquad k = 1, 2, \ldots \end{aligned}$$

Putting

$$\alpha_k(v) = P_k(v) - \Lambda_k(v), \qquad \beta_k(v) = \pi_k(v) - P_k(v), \qquad k = 0, 1, 2, \ldots$$

we introduce the generating functions

$$A_z(v) = \sum_{k=0}^{\infty} \alpha_k(v) z^k, \qquad B_z(v) = \sum_{k=0}^{\infty} \beta_k(v) z^k.$$

Using (12) we obtain the differential equation

$$(13) \qquad \frac{dA_z(v)}{dv} = (z-1)A_z(v) + (z-1)B_z(v), \qquad A_z(0) = 0.$$

Resolving this equation we have

$$(14) \qquad A_z(v) = (z-1)e^{(z-1)v} \int_0^v e^{-(z-1)u} B_z(u)\,du.$$

Let D stand for the differentiation operation with respect to z. Since

$$\frac{1}{k!} D^{(k)} A_z(v)\big|_{z=0} = P_k(v) - \Lambda_k(v),$$

we obtain

$$\sum_{k=0}^{\infty} |P_k(v) - A_k(v)| = \sum_{k=0}^{\infty} \frac{1}{k!} \left| D^{(k)} A_z(v)|_{z=0} \right|.$$

Further, we get

$$D^{(k)} A_z(v) = D^{(k)} [(z-1) e^{(z-1)v} \int_0^v e^{(1-z)u} B_z(u)\, du]$$

$$= e^{-v} \int_0^v e^u D^{(k)} [e^{z(v-u)} (z-1) B_z(u)]\, du.$$

Using the formula

$$D^{(k)} e^{\lambda z} u(z) = e^{\lambda z} (D+\lambda)^{(k)} u(z),$$

we can write

$$D^{(k)} A_z(v) = e^{-v} \int_0^v e^u e^{z(v-u)} (D+v-u)^{(k)} [(z-1) B_z(u)]\, du$$

$$= e^{-v} \int_0^v e^u e^{z(v-u)} \sum_{j=0}^{k} \frac{k!}{j!(k-j)!} (v-u)^j D^{(k-j)} [(z-1) B_z(u)]\, du.$$

Hence

$$\text{(15)} \qquad \left. \frac{D^{(k)} A_z(v)}{k!} \right|_{z=0} = e^{-v} \int_0^v e^u \sum_{j=0}^{k} \frac{(v-u)^j}{j!} \left. \frac{D^{(k-j)} (z-1) B_z(u)}{(k-j)!} \right|_{z=0} du.$$

Further, we obtain

$$\text{(16)} \qquad D^{(k-j)} (z-1) B_z(u)|_{z=0} = D^{(k-j)} z B_z(u)|_{z=0} - D^{(k-j)} B_z(u)|_{z=0}$$

$$= (k-j)! \left(\beta_{k-j-1}(u) - \beta_{k-j}(u)\right), \qquad k = 0, 1, 2, \ldots, \; j = 0, 1, \ldots, k,$$

where we put $\beta_{-1}(u) \equiv 0$. Substituting (16) in (15) we have

$$\left. \frac{D^{(k)} A_z(v)}{k!} \right|_{z=0} = e^{-v} \int_0^v e^u \sum_{j=0}^{k} \frac{(v-u)^j}{j!} \left(\beta_{k-j-1}(u) - \beta_{k-j}(u)\right) du.$$

Hence

$$\sum_{k=0}^{\infty} |P_k(v) - A_k(v)| = \sum_{k=0}^{\infty} \left| \frac{D^{(k)} A_z(v)}{k!} \right|_{z=0}$$

$$= e^{-v} \sum_{k=0}^{\infty} \left| \int_0^v e^u \sum_{j=0}^{k} \frac{(v-u)^j}{j!} \left(\beta_{k-j}(u) - \beta_{k-j-1}(u)\right) du \right|.$$

We write

$$\sum_{j=0}^{k} \frac{(v-u)^j}{j!} \left(\beta_{k-j}(u) - \beta_{k-j-1}(u)\right) = \sum_{j=0}^{k} \left(\frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right) \beta_{k-j}(u),$$

where, by definition, $(v-u)^{-1}/(-1)! \equiv 0$. Therefore, we get

$$\sum_{k=0}^{\infty} |P_k(v)-\Lambda_k(v)| = e^{-v} \sum_{k=0}^{\infty} \left| \int_0^v e^u \sum_{j=0}^{k} \left(\frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right) \beta_{k-j}(u)\, du \right|$$

$$\leqslant e^{-v} \sum_{k=0}^{\infty} \int_0^v e^u \sum_{j=0}^{k} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| |\beta_{k-j}(u)|\, du$$

$$\leqslant e^{-v} \int_0^v e^u \sum_{j=0}^{\infty} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| \varrho(u)\, du$$

$$= e^{-v} \int_0^{v_0} e^u \sum_{j=0}^{\infty} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| \varrho(u)\, du +$$

$$+ e^{-v} \int_{v_0}^{v} e^u \sum_{j=0}^{\infty} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| \varrho(u)\, du, \quad 0 < v_0 < v.$$

Consequently, we obtain

$$e^{-v} \int_{v_0}^{v} e^u \sum_{j=0}^{\infty} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| \varrho(u)\, du \leqslant 2 \int_{v_0}^{\infty} \varrho(u)\, du$$

and

$$e^{-v} \int_0^{v_0} e^u \sum_{j=0}^{\infty} \left| \frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right| \varrho(u)\, du$$

$$= e^{-v} \int_0^{v_0} e^u \sum_{j=0}^{[v-u]} \left(\frac{(v-u)^j}{j!} - \frac{(v-u)^{j-1}}{(j-1)!} \right) \varrho(u)\, du +$$

$$+ e^{-v} \int_0^{v_0} e^u \sum_{j=[v-u]+1}^{\infty} \left(\frac{(v-u)^{j-1}}{(j-1)!} - \frac{(v-u)^j}{j!} \right) \varrho(u)\, du$$

$$\leqslant c \int_0^{v_0} e^{u-v} \frac{(v-u)^{[v-u]}}{[v-u]!} \varrho(u)\, du \leqslant \frac{\hat{c}}{\sqrt{[v-v_0]}} \int_0^{\infty} \varrho(u)\, du, \qquad 0 < c,\ \hat{c} < \infty.$$

In the last inequality we applied Stirling's formula. Finally, we get

$$\sum_{k=0}^{\infty} |P_k(v)-\Lambda_k(v)| \leqslant \int_{v_0}^{\infty} \varrho(u)\, du + \frac{\hat{c}}{\sqrt{[v-v_0]}} \int_0^{\infty} \varrho(u)\, du.$$

By choosing v_0 and v sufficiently large the last expression can be made arbitrarily small. Since Theorem 2 is true for the Poisson distribution (see [5]), the proof is complete.

6. Proof of Theorem 3. It is sufficient to show that

$$\sup_k |\sqrt{v}\, P_k(v) - \sqrt{v}\, \Lambda_k(v)| \to 0 \quad \text{as } v \to \infty. \tag{17}$$

By (17) and the local limit theorem for the Poisson distribution, Theorem 3 holds.

By the converse formula for the Fourier transformation of the sequences $\alpha_k(v)$ and (14) we have

$$P_k(v)-\Lambda_k(v)=\frac{1}{2\pi}\int_{-\pi}^{\pi} A_{\exp\{it\}}(v)\,e^{-itk}\,dt$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}(e^{it}-1)\exp\{v(e^{it}-1)\}\Big[\int_0^v \exp\{(1-e^{it})u\}\,B_{\exp\{it\}}(u)\,du\Big]\,dt.$$

Replacing t by $t/\sqrt{v}$, we get

$$\sqrt{v}\,P_k(v)-\sqrt{v}\,\Lambda_k(v)=\frac{1}{2\pi}\int_{-\pi\sqrt{v}}^{\pi\sqrt{v}}\left(\exp\left\{\frac{it}{\sqrt{v}}\right\}-1\right)\times$$

$$\times\exp\left\{t\left[\exp\left(\frac{it}{\sqrt{v}}\right)-1\right]\right\}\left[\int_0^v \exp\left\{\left[1-\exp\left(\frac{it}{\sqrt{v}}\right)\right]u\right\}B_{\exp\{it/\sqrt{v}\}}(u)\,du\right]dt.$$

Further, we obtain

$$(18)\qquad |\sqrt{v}\,P_k(v)-\sqrt{v}\,\Lambda_k(v)|$$

$$\leqslant\frac{1}{2\pi}\int_{-\pi\sqrt{v}}^{\pi\sqrt{v}}\left|\exp\left\{\frac{it}{\sqrt{v}}\right\}-1\right|\left[\int_0^v\left|\exp\left\{\left[\exp\left(\frac{it}{\sqrt{v}}\right)-1\right](v-u)\right\}\right|\varrho(u)\,du\right]dt$$

$$\leqslant\frac{1}{\pi\sqrt{v}}\int_0^v\varrho(u)\left[\int_0^{\pi\sqrt{v}} t\exp\left\{-2(v-u)\sin^2\frac{t}{2\sqrt{v}}\right\}dt\right]du$$

$$=\frac{1}{\pi\sqrt{v}}\int_0^v\varrho(u)\left[\int_0^{\pi\sqrt{v}\varepsilon} t\exp\left\{-2(v-u)\sin^2\frac{t}{2\sqrt{v}}\right\}dt\right]du+$$

$$+\frac{1}{\pi\sqrt{v}}\int_0^v\varrho(u)\left[\int_{\pi\sqrt{v}\varepsilon}^{\pi\sqrt{v}} t\exp\left\{-2(v-u)\sin^2\frac{t}{2\sqrt{v}}\right\}dt\right]du,\qquad 0<\varepsilon<1.$$

We now estimate the first integral in (18). Since $0<t/2\sqrt{v}<\pi\varepsilon/2$, choosing $\varepsilon>0$ sufficiently small we can find $\alpha>0$ such that $\sin(t/2\sqrt{v})>\alpha t/2\sqrt{v}$. Therefore, we obtain

$$\frac{1}{\pi\sqrt{v}}\int_0^v\varrho(u)\left[\int_0^{\pi\sqrt{v}\varepsilon} t\exp\left\{-2(v-u)\sin^2\frac{t}{2\sqrt{v}}\right\}dt\right]du$$

$$\leqslant\frac{1}{\pi\sqrt{v}}\int_0^v\varrho(u)\left[\int_0^{\pi\sqrt{v}\varepsilon} t\exp\left\{-\alpha\frac{t^2}{2v}(v-u)\right\}dt\right]du$$

$$=\frac{1}{\alpha\pi\sqrt{v}}\int_0^v\frac{v}{v-u}\varrho(u)\left(1-\exp\left\{-\frac{\alpha\pi^2\varepsilon^2(v-u)}{2}\right\}\right)du$$

$$=\frac{1}{\alpha\pi\sqrt{v}}\int_0^v\varrho(u)\left(1-\exp\left\{-\frac{\alpha\pi^2\varepsilon^2(v-u)}{2}\right\}\right)du+$$

$$+\frac{1}{\alpha\pi\sqrt{v}}\int_0^v \frac{u}{v-u}\varrho(u)\left(1-\exp\left\{-\frac{\alpha\pi^2\varepsilon^2(v-u)}{2}\right\}\right)du$$

$$\leqslant \frac{c_1}{\sqrt{v}}\int_0^v \varrho(u)\,du+\frac{c_2}{\sqrt{v}}\int_0^v u\varrho(u)\,du, \qquad 0<c_1,\ c_2<\infty.$$

Hence the first integral in (18) tends to zero as $v \to \infty$.

We now estimate the second integral in (18). Clearly, we have

$$\exp\left\{-2\sin^2\frac{t}{2\sqrt{v}}\right\}<e^{-c}, \qquad \pi\sqrt{v}\varepsilon<t<\pi\sqrt{v},\ 0<\varepsilon<1,\ 0<c<\infty.$$

Therefore, we obtain

$$\frac{1}{\pi\sqrt{v}}\int_0^v \varrho(u)\left[\int_{\pi\sqrt{v}\varepsilon}^{\pi\sqrt{v}} t\exp\left\{-2(v-u)\sin^2\frac{t}{2\sqrt{v}}\right\}dt\right]du$$

$$\leqslant \frac{1}{\pi\sqrt{v}}\int_0^v \varrho(u)\left[\int_{\pi\sqrt{v}\varepsilon}^{\pi\sqrt{v}} te^{-c(v-u)}\,dt\right]du=\frac{c_3 v}{\pi\sqrt{v}}\int_0^v e^{-c(v-u)}\varrho(u)\,du$$

$$=c_4\sqrt{v}\,e^{-cv}\int_0^{v\gamma} e^{cu}\varrho(u)\,du+c_4\sqrt{v}\,e^{-cv}\int_{v\gamma}^{v} e^{cu}\varrho(u)\,du,$$

$$0<c_3,\ c_4<\infty,\ 0<\gamma<1.$$

We complete the proof of the theorem observing that

$$\sqrt{v}\,e^{-cv}\int_0^{v\gamma} e^{cu}\varrho(u)\,du \leqslant \sqrt{v}\,e^{-cv}e^{cv\gamma}\int_0^{v\gamma}\varrho(u)\,du$$

$$=ve^{-cv(1-\gamma)}\frac{1}{\sqrt{v}}\int_0^v \varrho(u)\,du \to 0 \qquad \text{as } v\to\infty$$

and

$$\sqrt{v}\,e^{-cv}\int_{v\gamma}^{v} e^{cu}\varrho(u)\,du \leqslant \sqrt{v}\int_{v\gamma}^{v}\varrho(u)\,du \leqslant \sqrt{v}\int_{v\gamma}^{v}\frac{u}{v\gamma}\varrho(u)\,du$$

$$\leqslant \frac{1}{\gamma\sqrt{v}}\int_0^v u\varrho(u)\,du \to 0 \qquad \text{as } v\to\infty.$$

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